香港國際數學競賽初賽 2019（香港賽區）

HONG KONG INTERNATIONAL MATHEMATICAL OLYMPIAD HEAT ROUND 2019 (HONG KONG REGION)

中學三年級 Secondary 3

時限：60 分鐘

Time allowed: 60 minutes

試題

Question Paper

考生須知:

Instructions to Contestants:

1. 本卷包括 試題 乙份，試題紙不可取走。

   Each contestant should have ONE Question-Answer Book which CANNOT be taken away.

2. 本卷共 5 個範疇，每範疇有 4 題，共 20 題，每題 3 分，總分 60 分，答錯不扣分。

   There are 5 exam areas and 4 questions in each exam area. There are a total of 20 questions in this Question-Answer Book. Each carries 3 marks. Total score is 60 marks. No points are deducted for incorrect answers.

3. 請將答案寫在 答題紙 上。

   All answers should be written on ANSWER SHEET.

4. 比賽期間，不得使用計算工具。

   NO calculators can be used during the contest.

5. 本卷中所有圖形不一定依比例繪成。

   All figures in the paper are not necessarily drawn to scale.

6. 比賽完畢時，本試題會被收回。

   This Question-Answer Book will be collected at the end of the contest.

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

填空題（第 1 至 20 題）（每題 3 分，答錯及空題不扣分）

Open-Ended Questions (1st ~20th) (3 points for correct answer, no penalty point for wrong answer)

<u>Logical Thinking</u>
<u>邏輯思維</u>

1. Given A, B and C are three non-zero distinct digits and the 3-digit numbers formed by these three digits have the following properties:
   1. $\overline{ACB}$ is divisible by 11;
   2. $\overline{ABC}$ is not divisible by 2;
   3. $\overline{CBA}$ is divisible by 4.
   4. $A > B > C > 1$
   Find the 3-digit number $\overline{BCB}$.

   已知 A、B 和 C 為三個非零而不同的數位且利用這三個數位組成的三位數有以下性質：
   1. $\overline{ACB}$ 可以被 11 整除；
   2. $\overline{ABC}$ 不可以被 2 整除；
   3. $\overline{CBA}$ 可以被 4 整除。
   4. $A > B > C > 1$
   求三位數 $\overline{BCB}$。

2. There are n lines that are not parallel with each other on a plane. There are no 3 lines intersecting at a point. If they intersect 66 times, find n.
   平面上有 n 條互不平行的線，且任何三線並不共點。若它們共有 66 個交點，求 n。

3. Alice goes southwest for 32km, then goes southeast for 21km, goes northeast for 20km and goes northwest for 16km. How far is she now from the original position?
   愛麗絲向西南走了 32 公里，向東南走了 21 公里，向東北走了 20 公里，向西北走了 16 公里，問她和原來位置相距多遠？

4. There are 15 problems in a competition. The scores of each problem are allocated in the following ways: 1 marks will be given for a correct answer, 1 marks will be deducted from a wrong answer and 0 marks will be given for a blank answer. Find the minimum number of candidate(s) to ensure that 3 candidates will have the same scores in the competition.
   某比賽共有 15 條題目。以下述方式為每個題目評分：答對得 1 分、答錯倒扣 1 分、不作答得零分。求最小參賽者的數目才可保證比賽中有三人同分。

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

Write down the answer in the simplest form. If the calculation result is a fraction, please write down the answer as a proper or mixed fraction, decimal figure is also accepted. Marks will NOT be given for incorrect unit.

Algebra
代數

5. If $x, y > 0, x^2 - y^2 - 27 = 6x + 12y$, find the value of $x - y$.

    若 $x, y > 0, x^2 - y^2 - 27 = 6x + 12y$，求 $x - y$ 的值。

6. Factorize $x^2 - 3y^2 - 2xy + 3x - y + 2$.

    因式分解 $x^2 - 3y^2 - 2xy + 3x - y + 2$。

7. Simplify $\sqrt{28 - 6\sqrt{3}}$.

    化簡 $\sqrt{28 - 6\sqrt{3}}$。

8. If $x^4 - 3x^3 + mx^2 - nx + 36$ is divisible by $x^2 - 5x + 6$, find mn.

    若 $x^4 - 3x^3 + mx^2 - nx + 36$ 能被 $x^2 - 5x + 6$ 整除，求 mn。

Number Theory
數論

9. Given that $\overline{8A2B1}$ is a 5-digit number which is divisible by 11, find the sum of possible value of $A - B$.

    已知 $\overline{8A2B1}$ 是一個五位數，且可被 11 整除，求 $A - B$ 的可能性之和。

10. Find the remainder for 268^{862} divided by 9.

    求 268^{862} 除以 9 的餘數。

11. Given that $11x \equiv 369 (\text{mod } 17)$, find the minimum value of $|x|$.

    已知 $11x \equiv 369 (\text{mod } 17)$，求 $|x|$ 的最小值。

12. Now is April. Which month will it be after 11^{13} months?

    現在是四月，11^{13} 個月後是幾月？

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。

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<u>Geometry</u>
幾何

13. Find the area enclosed by the x-axis, y-axis and the straight line $y = 2 - \dfrac{2}{7}x$.

    求由 x-軸、y-軸及直線 $y = 2 - \dfrac{2}{7}x$ 圍出的面積。

14. Find the minimum value of $\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$.
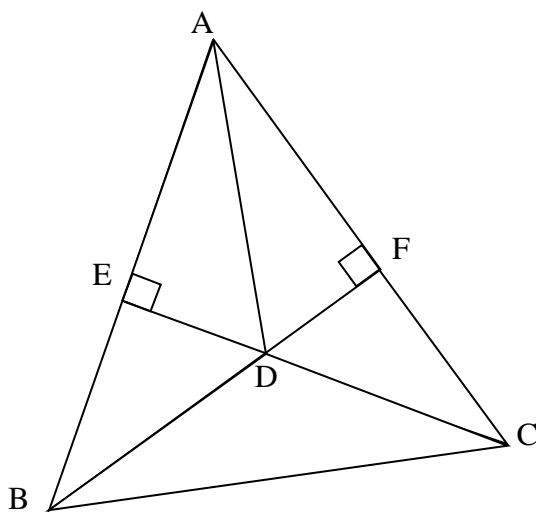
    求 $\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$ 的最小值。

15. Given that an interior angle of an n-sided regular polygon is 30 degrees more than $\dfrac{n}{4}$ times of an exterior angle. Find the value of n.

    已知一正 n 邊形其中一個內角比一個外角的 $\dfrac{n}{4}$ 倍大 30 度。求 n 的值。

16. Refer to the figure below, $DE:AD = 1:5$, $AB \perp CE$, $AC \perp BF$. Find the value of $\dfrac{AB + AC}{BC}$.

    參考附圖，$DE:AD = 1:5$，$AB \perp CE$，$AC \perp BF$。求 $\dfrac{AB + AC}{BC}$ 的值。

Question 16

第 16 題

Combinatorics
組合數學

17. In how many possible ways can 12 identical flowers be distributed to 3 distinct vases with at most 1 vase has no flower?
把 12 支完全相同的花放入三個不同的花瓶中，最多一個花瓶沒有花，問共有多少個不同的分配方法?

18. There are 4 identical blue boxes, 3 identical red boxes and 2 identical white boxes are put from left to right. How many way(s) of arrangement is / are there?
由左至右放置 4 個相同的藍色箱子，3 個相同的紅色箱子和 2 個相同的白色箱子，有多少個不同排列方法?

19. If x and y are positive integers such that $5x + 9y = 148$. Find the maximum value of $x + y$.
已知 x、y 為正整數且 $5x + 9y = 148$，求 $x + y$ 的最大值。

20. Given (a, b, c, d) is a set of integers and all of them are greater than -2. Find the number of solution set(s) of $a + b + c + d = 5$.
已知 (a, b, c, d) 為一組整數且全部大於 -2，求方程 $a + b + c + d = 5$ 的解組數目。

~ 全卷完 ~
~ End of Paper ~

請以最簡形式填寫答案，若計算結果是分數，請確保為真分數或帶分數，或將計算結果寫成小數。錯誤單位將不給予任何分數。
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